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Nonlocal effects on D-branes in plane-wave backgrounds

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Abstract: We argue that the effective field theory on D3-branes in a plane-wave background with 3-form flux is a nonlocal deformation of Yang-Mills theory. In the case of NSNS flux, it is a dipole field theory with lightlike dipole vectors. For an RR 3-form flux the dipole theory is strongly coupled. We propose a weakly coupled S-dual description for it. The S-dual description is local at any finite order in string perturbation theory but becomes nonlocal when all perturbation theory orders are summed together.

Keywords: Duality in Gauge Field Theories, D-branes, Penrose limit and pp-wave background.

Contents

1. Introduction

The restrictions imposed by the conditions of Lorentz invariance and locality play central roles in our understanding of the formal properties of quantum field theories. However, in string theory neither of these conditions appears to be fundamental. Thus, it is interesting to consider simple situations where they are relaxed. In particular, we will examine the properties of D-branes in certain plane-wave backgrounds with strong 3-form fields. As we will show in detail, the low energy effective theory describing the fluctuations of these D-branes is a non-local, Lorentz violating dipole theory [1]-[3].

Typical interaction terms in the lagrangian of this field theory are of the form

$$\int \phi_1(\vec{x})\phi_2(\vec{x}+\vec{L}_1)\phi_3(x+\vec{L}_1+\vec{L}_2)\cdots d^4\vec{x}$$

where ϕ_i are fields and the $\vec{L}_i$ are fixed world-volume vectors. Roughly speaking, the non-locally coupled fields ϕ_i correspond to stretched open strings with end-points that are separated by $\vec{L}_i$ and with angular momentum along planes transverse to the brane. These strings are stabilized by the presence of strong 3-form fluxes with legs aligned along the dipole vectors as well as the plane of rotation [2].

An exciting application of string theory with strong 3-form field strengths is the AdS_3/CFT_2 correspondence [4]. Unfortunately, progress had been limited by the fact that string theory in AdS backgrounds with RR field strengths are difficult to analyze exactly. However, the authors of [5] have shown that a particularly tractable limit of the AdS/CFT correspondence can be obtained by taking the Penrose limit of type-IIB string theory on $AdS_5 \times S^5$ to obtain a plane-wave background. They were able to precisely match the properties of a certain subsector of $\mathcal{N} = 4$ Super-Yang-Mills CFT (operators with large R-charge) with the exact results of [6]–[8] for strings in plane-wave backgrounds.

Similarly, one can consider the Penrose limits of $AdS_3 \times S^3 \times T^4$ [9, 10]. As IIB has two three-form field strengths, H^1 (NSNS) and H^2 (RR), one finds a pair of models which are related by S-duality. The Penrose limit of the theory with H^1 flux is

$$\begin{aligned} ds^2 &= dx^+dx^- + \mu x^i x^i (dx^+)^2 - dx^a dx^a - dx^i dx^i, && (1.1)\\ H^1 &= -\mu dx^+ \wedge (dx^6 \wedge dx^7 + dx^8 \wedge dx^9), && (1.2)\\ e^\phi &= g_s, && (1.3) \end{aligned}$$

where ds^2 is the interval in string frame, $x^\pm = x^0 \pm x^1$, the x^a are coordinates on T^4 with $a = 2, \ldots, 5$ and $i = 6, \ldots, 9$. The Penrose limit of the S-dual configuration is

$$\begin{aligned} ds^2 &= dx^+dx^- + \mu x^i x^i (dx^+)^2 - dx^a dx^a - dx^i dx^i, && (1.4)\\ H^2 &= \mu dx^+ \wedge (dx^6 \wedge dx^7 + dx^8 \wedge dx^9), && (1.5)\\ e^\phi &= \frac{1}{g_s}. && (1.6) \end{aligned}$$

Exact results for the spectrum of both models were obtained in [9, 10]. Further, open strings and D-branes in these and other plane-wave backgrounds have been studied in [11]–[23].

In this paper we will study the interactions of the low energy effective theory of the D-brane excitations. We will show that N D3-brane probes of the plane-wave background (1.1)–(1.3) are exactly described at low energies by a nonlocal U(N) dipole gauge theory [1] with a lightlike dipole vector $\vec{L}$ proportional to μ.

A more complicated problem is the description of N D3-brane probes of the pp-wave background (1.4)–(1.6), which has RR flux. It is related to the S-dual description of the lightlike dipole theory. We attack this problem by first studying the S-dual description of a U(1) lightlike dipole theory and then guessing the generalization of that result to a U(N) gauge group. We find that in any finite order of string perturbation theory the interactions of the D3-brane probes of the pp-wave background (1.4)–(1.6) are local. Yet our result suggests that summing the local interactions to all orders in perturbation theory exhibits an intrinsic nonlocality with a characteristic length proportional to the string coupling constant, g_s.

The paper is organized as follows. In section 2 we review the definition and salient features of dipole theories. In section 3 we identify the lightlike dipole theory as the low energy description of D3-branes in the pp-wave background (1.1)–(1.3). In section 4 we analyze the S-dual of the U(1) lightlike dipole theories and conjecture an extension of the result to U(N), We conclude in section 5 with a list of possible extensions of our work.

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2. Definition of dipole theories and their salient features

The dipole field theories that we will work with in this paper are nonlocal field theories that are deformations of $\mathcal{N}=4$ SYM. The lagrangian of $\mathcal{N}=4$ SYM is

$$\begin{aligned}
\mathcal{L}_{\mathcal{N}=4} &= \frac{1}{g^2}\mathrm{tr}\left\{\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}\sum_{I=1}^{6}D_\mu\Phi^I D^\mu\Phi^I + i\sum_{a=1}^{4}\overline{\psi}^{\dot\alpha}_a\sigma^{\mu\alpha}{}_{\dot\alpha}D_\mu\psi^a_\alpha\right\}+\\
&+\frac{1}{g^2}\mathrm{tr}\left\{\sum_{I<J}\left[\Phi^I,\Phi^J\right]^2+\epsilon^{\alpha\beta}\sum_{I,a,b}\gamma^I_{ab}\Phi^I\psi^a_\alpha\psi^b_\beta+\epsilon_{\dot\alpha\dot\beta}\sum_{I,a,b}\gamma^{Iab}\Phi^I\overline{\psi}^{\dot\alpha}_a\overline{\psi}^{\dot\beta}_b\right\},\\
D_\mu\Phi^I &\equiv \partial_\mu\Phi^I + i\left[A_\mu,\Phi^I\right].
\end{aligned} \tag{2.1}$$

Here Φ^I ($I=1,\ldots,6$) are adjoint scalar fields of $\mathrm{U}(N)$ which transform as a vector of the R-symmetry group Spin(6). The ψ^a_α ($a=1,\ldots,4$) are adjoint Weyl fermions in the **4** of Spin(6). Their complex conjugate fields $\overline{\psi}^{\dot\alpha}_a$ transform in the complex conjugate representation $\overline{\mathbf{4}}$ of Spin(6). γ^{Iab} are the Clebsch-Gordan coefficients of Spin(6) and $\sigma^{\mu\alpha}{}_{\dot\alpha}$ are Pauli matrices.

The dipole theories are obtained from $\mathcal{N}=4$ SYM by the following steps (see [3] for more details):

1. Define the complex linear combinations of the 6 scalar fields of (2.1):

$$Z_k \equiv \Phi_{2k-1}+i\Phi_{2k}\,,\qquad \overline{Z}_k \equiv \Phi_{2k-1}-i\Phi_{2k}\,,\qquad k=1,2,3\,,$$

   and assign a constant space-time 4-vector $\vec{L}_k$ to each scalar field Z_k.

2. Modify the covariant derivatives of the scalar fields so that $D_\mu Z_k$ at the space-time point x will be:

$$D_\mu Z_k(x)\equiv\partial_\mu Z_k(x)-iA_\mu\left(x-\frac{1}{2}\vec{L}_k\right)Z_k(x)+iZ_k(x)A_\mu\left(x+\frac{1}{2}\vec{L}_k\right). \tag{2.2}$$

   Note that the fields Z_k are $N\times N$ matrices in the adjoint representation of $\mathrm{U}(N)$. Thus, equation (2.2) implies that the quanta of the fields Z_k are dipoles whose ends are at $x\pm\frac{1}{2}\vec{L}_k$. The gauge transformation of the scalar fields is

$$Z_k(x)\mapsto\Omega^{-1}\left(x-\frac{1}{2}\vec{L}_k\right)Z_k(x)\Omega\left(x+\frac{1}{2}\vec{L}_k\right),$$

   where $\Omega(x)\in\mathrm{U}(N)$ is the gauge group element.

3. In order to preserve $\mathrm{U}(N)$ gauge invariance we have to modify the definition of the commutators in (2.1) to:

$$[Z_k,Z_l]_{(x)}\to Z_k\left(x-\frac{1}{2}\vec{L}_l\right)Z_l\left(x+\frac{1}{2}\vec{L}_k\right)-Z_l\left(x-\frac{1}{2}\vec{L}_k\right)Z_k\left(x+\frac{1}{2}\vec{L}_l\right).$$

4. We also need to modify the interactions of the fermions with the scalars so as to be gauge invariant. This can be done by assigning to the fermions their own dipole-vectors. To find the appropriate assignment we need to correlate the dipole-vector of the various fields with their Spin(6) = SU(4) R-symmetry charges, as follows. The parameters $\vec{L}_k$ that define the dipole theory can be combined into a single linear map $\Upsilon : su(4) \to \mathbb{R}^{3,1}$ from the Lie algebra of the R-symmetry group to a spacetime 4-vector. Using the inner product on $su(4)$, Υ can be represented as an $su(4)$-valued spacetime 4-vector. In the representation **6** of $su(4)$ we can take Υ to be

$$\Upsilon \xrightarrow{\mathbf{6}} \begin{pmatrix} 0 & \vec{L}_1 & 0 & 0 & 0 & 0 \\ -\vec{L}_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \vec{L}_2 & 0 & 0 \\ 0 & 0 & -\vec{L}_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \vec{L}_3 \\ 0 & 0 & 0 & 0 & -\vec{L}_3 & 0 \end{pmatrix}. \tag{2.3}$$

Now we can define the interactions of the fermions. We need to write Υ in the representation **4** of $su(4)$ and find a basis of this representation where Υ is diagonal. It will then have the following form:

$$\Upsilon \xrightarrow{\mathbf{4}} \begin{pmatrix} \vec{\lambda}_1 & 0 & 0 & 0 \\ 0 & \vec{\lambda}_2 & 0 & 0 \\ 0 & 0 & \vec{\lambda}_3 & 0 \\ 0 & 0 & 0 & \vec{\lambda}_4 \end{pmatrix},$$

with the definitions

$$\begin{aligned} \vec{\lambda}_1 &= \frac{1}{2}(\vec{L}_1 + \vec{L}_2 + \vec{L}_3), \\ \vec{\lambda}_2 &= \frac{1}{2}(\vec{L}_1 - \vec{L}_2 - \vec{L}_3), \\ \vec{\lambda}_3 &= \frac{1}{2}(-\vec{L}_1 + \vec{L}_2 - \vec{L}_3), \\ \vec{\lambda}_4 &= \frac{1}{2}(-\vec{L}_1 - \vec{L}_2 + \vec{L}_3). \end{aligned} \tag{2.4}$$

The Weyl fermions ψ^a_α ($a = 1, \ldots, 4$) of (2.1), which are in the **4** of $su(4)$, should be assigned the dipole vectors $\vec{\lambda}^a$ and their complex conjugate fields should be assigned $(-\vec{\lambda}^a)$. To get a gauge invariant lagrangian we need to replace all the commutators of a scalar and a fermion with:

$$[Z_k, \psi^a]_{(x)} \to Z_k\left(x - \frac{1}{2}\vec{\lambda}_a\right)\psi\left(x + \frac{1}{2}\vec{L}_k\right) - \psi\left(x - \frac{1}{2}\vec{L}_k\right)Z_k\left(x + \frac{1}{2}\vec{\lambda}_a\right).$$

5. Since the gauge bosons have vanishing dipole vectors, preserving any supersymmetry requires that some of the fermions have vanishing dipole vectors [3]. In particular, to preserve $\mathcal{N} = 2$ we may choose $\vec{\lambda}_1 = -\vec{\lambda}_2 = \vec{L}_2 = \vec{L}_3 = \vec{L}$ and $\vec{\lambda}_3 = \vec{\lambda}_4 = \vec{L}_1 = 0$.

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These rules can be recast as a redefinition of the product of two fields. The modified product of any two fields $\Xi_1(x), \Xi_2(x)$ (scalar, fermionic or gauge) is defined in a way somewhat reminiscent of noncommutative geometry [24, 25]:

$$(\Xi_1 * \Xi_2)_{(x)} \equiv e^{\frac{i}{2}\langle\Upsilon^\mu,\hat{R}_1\rangle\frac{\partial}{\partial z_\mu} - \frac{i}{2}\langle\Upsilon^\mu,\hat{R}_2\rangle\frac{\partial}{\partial y_\mu}} (\Xi_1(y)\Xi_2(z))\Big|_{y=z=x}, \tag{2.5}$$

where $\hat{R}_i$ $(i = 1, 2)$ is the ($su(4)$-valued) R-symmetry charge operator acting on Ξ_i and $\langle\cdot,\cdot\rangle$ is the Killing form on $su(4)$ (see [3] for more details).

Special cases of dipole theories have been discussed in [26, 27] and various aspects of the theories have been explored in [28]–[33].

Lightlike dipole-vectors. Define the linear vector space $W \subset \mathbb{R}^{3,1}$ to be the image of the map $\Upsilon : su(4) \to \mathbb{R}^{3,1}$ defined in (2.3). In terms of the fundamental dipole vectors that were introduced in (2.3):

$$W = \mathrm{Span}\{\vec{L}_1, \vec{L}_2, \vec{L}_3\}.$$

We will define the dipole theory to be *lightlike* if W is 1-dimensional and null, i.e.

$$\vec{L}_i \cdot \vec{L}_j = 0, \qquad i, j = 1, 2, 3.$$

As we shall see in section 4, lightlike dipole theories are easier to analyze than the generic dipole theories. This is similar to Yang-Mills theory on a noncommutative space that simplifies when the noncommutativity parameter is lightlike [34]. Lightlike deformation parameters have also been used in the context of the noncommutative (2,0)-theory [35]–[37].

3. Lightlike dipole theories and NSNS plane-wave backgrounds

In this section we will show that the low energy effective actions describing appropriately oriented D3-branes in a plane-wave background with a strong lightlike NSNS 3-form flux are lightlike dipole theories. The orientation of the D3-branes must be such that, in the notation of (1.1)–(1.3), the $+, -$ directions are longitudinal and the x^i $(i = 6, \ldots, 9)$ directions are transverse.

3.1 Geometric engineering of dipole-theories

To obtain a lightlike dipole theory we consider a background in which probe D3 branes have a small timelike dipole vector and then we perform a large boost. For simplicity, assume that all the dipole vectors which are encoded in Υ are in the x^1 direction. In this case Υ reduces to a single element in the Lie algebra $su(4)$ which, in the representation **6**, we can write as a 6×6 antisymmetric matrix $2\pi\alpha'\hat{Q}$.

As was shown in [3], a U(N) dipole theory with dipole vectors along x^1 described by $2\pi\alpha'\hat{Q}$ arises as the low-energy effective action of N D3-brane probes in the string theory background,

$$\begin{aligned} ds^2 &= dt^2 - \frac{1}{1+\vec{x}^\top \hat{Q}^\top \hat{Q}\vec{x}}(dx^1)^2 - (dx^2)^2 - (dx^3)^2 - d\vec{x}^\top d\vec{x} + \frac{(d\vec{x}^\top \hat{Q}\vec{x})^2}{1+\vec{x}^\top \hat{Q}^\top \hat{Q}\vec{x}} \\ B &= \frac{1}{2}\frac{d\vec{x}^\top \hat{Q}\vec{x}}{1+\vec{x}^\top \hat{Q}^\top \hat{Q}\vec{x}}\wedge dx^1\,, \qquad e^{2(\phi-\phi_0)} = \frac{1}{1+\vec{x}^\top \hat{Q}^\top \hat{Q}\vec{x}}\,, \end{aligned}$$

where $\vec{x} = (x^4, \ldots, x^9)$. We can obtain a theory with a lightlike dipole vector by infinitely boosting this background along x^1. As the dipole vector prior to the boost has a magnitude set by $2\pi\alpha'\hat{Q}$, we must simultaneously scale $\hat{Q} \to 0$ to obtain a lightlike dipole vector which has finite components in this limit. Thus, let

$$x^1 = \gamma(x^{1'} + vt')\,, \qquad t = \gamma(t' + vx^{1'})\,, \qquad \gamma \equiv \frac{1}{\sqrt{1-v^2}}$$

and take $v \to 1$ while keeping

$$\gamma\hat{Q} \equiv Q = \text{finite}\,.$$

Defining $x^\pm \equiv t' \pm x^{1'}$ we find the background

$$\begin{aligned} ds^2 &= dx^+dx^- + (\vec{x}^\top Q^\top Q\vec{x})(dx^+)^2 - (dx^2)^2 - (dx^3)^2 - d\vec{x}^\top d\vec{x} \\ B &= \frac{1}{2}d\vec{x}^\top Q\vec{x}\wedge dx^+\,, \qquad e^\phi = g_s\,. \end{aligned} \tag{3.1}$$

In order to preserve $\mathcal{N} = 2$ supersymmetry, we take

$$2\pi\alpha' Q = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L^- & 0 & 0 \\ 0 & 0 & -L^- & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & L^- \\ 0 & 0 & 0 & 0 & -L^- & 0 \end{pmatrix}. \tag{3.2}$$

Concretely, we note that the dipole vectors for the fields in this background are of the form $\vec{L} = (L^-, \pm L^-, 0, 0)$. If we define

$$\mu \equiv \frac{L^-}{2\pi\alpha'}\,, \tag{3.3}$$

we see that this background (3.1) is exactly the NSNS plane-wave of equation (1.1)–(1.3),

$$\begin{aligned} ds^2 &= dx^+dx^- + \mu x^i x^i (dx^+)^2 - dx^a dx^a - dx^i dx^i\,, \\ H^1 &= -\mu dx^+ \wedge (dx^6\wedge dx^7 + dx^8\wedge dx^9)\,, \\ e^\phi &= g_s\,, \end{aligned} \tag{3.4}$$

where again $a = 2, \ldots, 5$ and $i = 6, \ldots, 9$.

Note that L^-, the characteristic length scale of nonlocality, can be made arbitrarily big by a coordinate transformation that rescales x^+. It is therefore obvious that the excited open string states decouple from the low energy lightlike dipole theory. Furthermore, since the lightlike dipole theory is a limit of a dipole theory with spacelike dipole vectors and since the latter can be constructed as a certain limit of compactified noncommutative $\mathcal{N} = 4$ Super Yang-Mills theory [1], it follows that the lightlike dipole theory is unitary.

3.2 Lightcone string theory in the NSNS background

Using the exact results of [5, 8] (extended by [21] to the open string case) for string theory in the NSNS plane-wave background (3.4), we will show directly that the open string interactions are modified by the phases one would expect for a lightlike dipole deformation.

In order to facilitate future comparisons to the RR case, we consider the plane wave background in the GS formalism. First, we define the complex worldsheet scalar fields

$$Z_1 \equiv X_6 + iX_7\,, \qquad Z_2 \equiv X_8 + iX_9\,.$$

In order to simplify the analysis of the interactions in lightcone gauge, it is conventional to fix $X^+ = p^+\tau$ and additionally require that the string length be $\ell = 2\pi\alpha' p^+$. Just as in [5, 8], we will find it useful to split our fermions into positive and negative chirality fermions with respect to Γ^{6789}. We use S to denote the positive chirality fermions. As the negative chirality fermions and the scalars X^a, $a = 2, \dots, 5$ remain free and massless, we will ignore them. The resulting action in lightcone gauge is then given by,

$$S = \frac{1}{2\pi\alpha'}\int d\tau \int_0^{2\pi\alpha' p^+} d\sigma \left[\frac{1}{2}\sum_{k=1}^{2}\left(|\dot{Z}_k|^2 - |Z_k' + i\mu Z_k|^2\right) + i\overline{S}\left(\sigma^0\partial_0 + \sigma^1(\partial_1 - \mu\Gamma^{67})\right)S\right]. \tag{3.5}$$

There exists a field redefinition that, locally in σ, transforms this action into that of a free string. This transformation is [8, 21]

$$\widetilde{Z}_k(\sigma) \equiv e^{i\mu\sigma} Z_k(\sigma)\,, \qquad \widetilde{S}(\sigma) \equiv e^{-\mu\Gamma^{67}\sigma} S(\sigma)\,. \tag{3.6}$$

In terms of the new fields the action is simply

$$S = \frac{1}{2\pi\alpha'}\int d\tau \int_0^{2\pi\alpha' p^+} d\sigma \left[\frac{1}{2}\sum_{k=1}^{2}\left(|\dot{\widetilde{Z}}_k|^2 - |\widetilde{Z}_k'|^2\right) + i\overline{\widetilde{S}}\left(\sigma^0\partial_0 + \sigma^1\partial_1\right)\widetilde{S}\right]. \tag{3.7}$$

Note that the transformation (3.6) can change the boundary conditions of various fields. For closed strings, the transformed fields no longer satisfy periodic boundary conditions and the closed string spectrum in the plane-wave background differs from that of the free string. However, the spectrum of open strings with Dirichlet boundary conditions is unaltered. Instead, the interactions are modified in an interesting way as we will discuss presently.

3.3 Lightlike dipole-theories on D-branes in a plane-wave background

Consider a D1-brane that is extended in the x^+, x^- directions. The extension of the discussion to D3-branes is straightforward. The open string excitations are described in lightcone gauge by the action (3.5) with the boundary conditions

$$Z_k(0) = Z_k(2\pi\alpha' p^+) = 0\,, \qquad 0 = S_L(0) - S_R(0) = S_L(2\pi\alpha' p^+) - S_R(2\pi\alpha' p^+)\,.$$

As the transformation (3.6) does not affect these boundary conditions, the spectrum of

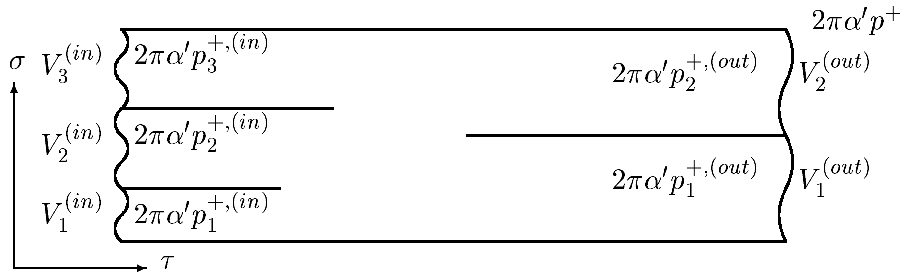

Figure 1: Scattering amplitude in the lightcone formalism.

Dirichlet-Dirichlet open strings ending on a D1-brane in this NSNS plane-wave background is the same as the flat space spectrum. The interactions, however, receive extra phases that precisely reproduce the interactions described in section 2.

Consider, for example, a tree level diagram that describes the scattering of open string states with vertex operators $V_1^{(in)}, \dots, V_{n_i}^{(in)}$ into open string states with vertex operators $V_1^{(out)}, \dots, V_{n_f}^{(out)}$ (see figure 1). When written in terms of $\widetilde{Z}_k$ and $\widetilde{S}$, these vertex operators should have the same form as the usual free Dirichlet-Dirichlet open string vertex operators. In fact, one might naively guess that as $\widetilde{Z}_k = e^{i\mu\sigma} Z_k$ the relation should be

$$V_j^{(in)}(Z_k(\sigma), \dots) = \widetilde{V}_j^{(in)}(e^{-i\mu\sigma} Z_k(\sigma), \dots)\,, \tag{3.8}$$

where $\widetilde{V}_j^{(in)}$ is the free string vertex operator that corresponds to the free string state with the same labels. This, of course, would give us the same amplitudes as in the free string case. However, note that if we let $p_j^{+,(in)}$ be the lightcone momentum of the j^{th} incoming string state, the parameter σ for that state is in the range

$$2\pi\alpha' \sum_{k=1}^{j-1} p_k^{+,(in)} \le \sigma \le 2\pi\alpha' \sum_{k=1}^{j} p_k^{+,(in)}\,,$$

which means that the prescription (3.8) for defining the vertex operator contains phase factors which depend on the position of the insertion of the operator along the string. This cannot be correct.

We can solve this problem by replacing σ with $\sigma' = \sigma - 2\pi\alpha' \sum_{k=1}^{j-1} p_k^{+,(in)}$ (which is the distance from the beginning of the j^{th} string) so

$$0 \le \sigma' \le 2\pi\alpha' p_j^{+,(in)} \tag{3.9}$$

$$V_j^{(in)}(Z_k(\sigma), \dots) = \widetilde{V}_j^{(in)}(e^{-i\mu\sigma'} Z_k(\sigma), \dots)\,. \tag{3.10}$$

This modification leads to overall phase shifts in the vertex operators as compared to the theory in flat space. To calculate them, we just need to know the Z_k and S dependence of the vertex operators. More formally, on the worldsheet there is a global U(1) symmetry which acts on the Z_k by $Z_k \to e^{i\theta} Z_k$ (and analogously on the fermions, which we neglect for simplicity). A general vertex operator will transform under this U(1) as $V^{(j)} \to e^{iq^{(j)}\theta} V^{(j)}$.

Noting that $\mu = L^-/2\pi\alpha'$, it is easy to see that the definitions (3.10) and (3.8) differ by the phase,

$$\exp\left\{ i \sum_{l=1}^{j-1} q^{(j)} L^- p_l^{+,(in)} \right\}.$$

If we let $p_r^{+,(out)}$ be the lightcone momentum of the r^{th} outgoing string state, momentum conservation requires $p^+ = \sum_{j=1}^{n_i} p_j^{+,(in)} = \sum_{r=1}^{n_f} p_r^{+,(in)}$ and we see that the overall phase for the entire amplitude is

$$\exp\left\{ i \sum_{j=1}^{n_i} \sum_{l=1}^{j-1} q^{j,(\mathrm{in})} L^- p_l^{+,(in)} - i \sum_{r=1}^{n_f} \sum_{s=1}^{r-1} q^{r,(\mathrm{out})} L^- p_s^{+,(out)} \right\}.$$

It is not hard to see that this is exactly the same phase as the one we get by Fourier expanding the Super Yang-Mills action of a D-brane and replacing every product with the modified $*$-product (2.5).

4. Proposal for the S-dual theory

In this section we will present our proposal for the S-dual of the lightlike dipole theories. We will begin with an analysis of a dipole theory with a U(1) gauge group and a single fermion (known as dipole QED [29, 32]) and then proceed to present our conjecture about a dipole theory with an SU(N) or U(N) gauge group.

The field contents of U(1) dipole QED (without any supersymmetry) is:

- A_μ the U(1) gauge field,
- ψ a Dirac fermion with dipole vector $\vec{L}$.

The lagrangian is

$$\mathcal{L} = \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2g^2} \overline{\psi} \gamma^\mu D_\mu \psi, \qquad D_\mu \psi \equiv \partial_\mu \psi - i \left[A_\mu \left(x + \frac{1}{2}\vec{L} \right) - A_\mu \left(x - \frac{1}{2}\vec{L} \right) \right] \psi. \tag{4.1}$$

Here $\vec{L}$ is the constant dipole-vector and we assume that it is spacelike or null.

As shown in [29, 32], the Feynman rules of this theory are identical to those of ordinary QED, with the following modification of the interaction vertex,

$$ig\gamma^\mu \to ig\gamma^\mu \times 2i \sin \frac{p \cdot L}{2}, \tag{4.2}$$

where p is the outgoing momentum of the photon. In particular, this means that the photon self-energy at one-loop just gets an extra factor of

$$2i \sin \frac{p \cdot L}{2} \times 2i \sin \frac{-p \cdot L}{2} = 4 \sin^2 \frac{p \cdot L}{2} \tag{4.3}$$

as compared to the QED result. This suggests that the U(1) theory is IR free, just like ordinary QED. Thus, our application of S-duality in the U(1) case will be somewhat formal, and should be considered simply as a motivation for the conjecture in the U(N) case.

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To find the S-dual description we will adopt the standard method of using a Lagrange multiplier for the field strength.[1] We treat $F_{\mu\nu}$ as an independent field subject to the Bianchi identity $\epsilon^{\mu\nu\tau\rho}\partial_\nu F_{\tau\rho} = 0$ which we implement with a Lagrange multiplier. Of course, this method requires that the gauge field A_μ does not appear explicitly in the lagrangian. Unlike in ordinary QED, here we can eliminate A_μ by performing a redefinition of variables [1]

$$\psi^{(inv)}(x) \equiv e^{-\frac{i}{2}\int_{-1}^{1} L^\mu A_\mu(x+\frac{s}{2}\vec{L})ds}\psi(x)\,, \tag{4.4}$$

so that $\psi^{(inv)}$ is a U(1)-neutral field. This is the analog of the Seiberg-Witten map [39] for dipole theories. Just as in that case, this transformation results in a theory with ordinary gauge symmetry perturbed by an infinite number of irrelevant interactions. In particular, since

$$D_\mu\psi(x) = e^{-\frac{i}{2}\int_{-1}^{1} L^\nu A_\nu(x+\frac{s}{2}\vec{L})ds}\left[\partial_\mu\psi^{(inv)}(x) + \frac{i}{2}\psi^{(inv)}(x)\int_{-1}^{1} L^\nu F_{\mu\nu}\left(x+\frac{s}{2}\vec{L}\right)ds\right],$$

we can define

$$D^F{}_\mu\psi^{(inv)}(x) \equiv \partial_\mu\psi^{(inv)}(x) + \frac{i}{2}\psi^{(inv)}(x)\int_{-1}^{1} L^\nu F_{\mu\nu}\left(x+\frac{s}{2}\vec{L}\right)ds\,,$$

to get the lagrangian

$$\mathcal{L}_1 = \frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2g^2}\overline{\psi}^{(inv)}\gamma^\mu D^F{}_\mu\psi^{(inv)}\,. \tag{4.5}$$

Thus, as promised, the explicit dependence on A_μ has been removed. Further, note that making the replacements $\psi \to g\psi$, and $A_\mu \to gA_\mu$ in the above lagrangian and writing

$$\int_{-1}^{1} ds F_{\mu\nu}\left(x+\frac{s}{2}\vec{L}\right) = \frac{2\sin\frac{i}{2}L\cdot\partial}{\frac{i}{2}L\cdot\partial}F_{\mu\nu}(x)\,, \tag{4.6}$$

we see that

$$\mathcal{L}_1 = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\overline{\psi}^{(inv)}\gamma^\mu\left(\partial_\mu + igL^\nu\frac{\sin\frac{i}{2}L\cdot\partial}{\frac{i}{2}L\cdot\partial}F_{\mu\nu}(x)\right)\psi^{(inv)}$$

is just a free theory perturbed by an infinite number of higher derivative interactions with couplings of the form $gL \times L^{2n}$.

We can now easily find the S-dual theory by treating $F_{\mu\nu}$ as an independent variable and adding a Lagrange multiplier to the lagrangian (4.5),

$$\mathcal{L}_2 = \frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2g^2}\overline{\psi}^{(inv)}\gamma^\mu D^F{}_\mu\psi^{(inv)} + \frac{1}{8\pi}\widetilde{A}_\mu\epsilon^{\mu\nu\tau\rho}\partial_\nu F_{\tau\rho}.$$

Except for the kinetic term, $F_{\mu\nu}$ appears linearly in $\mathcal{L}_2$. Thus, we can integrate it out to get,

$$\mathcal{L}' = \frac{1}{4g'^2}\left(\widetilde{F}_{\mu\nu} - \frac{1}{\pi g'^2}\epsilon_{\mu\nu\tau\rho}L^\tau\int_{-1}^{1}\widetilde{J}^\rho\left(x+\frac{s}{2}\vec{L}\right)ds\right)^2 + \frac{1}{2g'^2}\overline{\widetilde{\psi}}\gamma^\mu\partial_\mu\widetilde{\psi}\,. \tag{4.7}$$

[1] A similar method was used in [38] to study S-duality for Super Yang-Mills theory on a noncommutative $\mathbb{R}^{3,1}$.

where we have defined

$$g' = \frac{4\pi}{g}\,, \qquad \widetilde{\psi} \equiv \frac{4\pi}{g^2}\psi\,, \qquad \widetilde{J}_\mu \equiv i\overline{\widetilde{\psi}}\gamma_\mu\widetilde{\psi}\,. \tag{4.8}$$

If we make a further redefinition of the fields,

$$\hat{\psi} \equiv \frac{1}{g'}\widetilde{\psi}\,, \qquad \hat{F} \equiv \frac{1}{g'}\widetilde{F}\,, \qquad \hat{L}^\tau \equiv \frac{1}{g'^2}L^\tau\,, \tag{4.9}$$

we see that (4.7) can be rewritten as

$$\mathcal{L}' = \frac{1}{4}\left(\hat{F}_{\mu\nu} - \frac{g'\hat{L}^\tau}{\pi}\epsilon_{\mu\nu\tau\rho}\int_{-1}^{1}\hat{J}^\rho\left(x + \frac{s}{2}g'^2\hat{L}\right)ds\right)^2 + \frac{1}{2}\overline{\hat{\psi}}\gamma^\mu\partial_\mu\hat{\psi}\,. \tag{4.10}$$

If we add minimally coupled scalars to the QED lagrangian (4.1), with the same dipole vector $\vec{L}$, the expression of the S-dual $\mathcal{L}'$ becomes more complicated because the interactions are quadratic in the gauge field. The dual lagrangian simplifies for a lightlike dipole vector. To see this, we will fix the QED lightcone gauge $A_+ = 0$. In this gauge the redefinition (4.4) becomes simply $\psi^{(inv)}(x) \equiv \psi(x)$. Following the same steps that led to (4.7) with minimally coupled scalars added we find that the dual lagrangian can be obtained from the QED lagrangian by the substitution

$$g \to g'\,, \qquad F_{\mu\nu}(x) \to F'_{\mu\nu}(x) \equiv F_{\mu\nu}(x) - \frac{\hat{L}^\tau}{\pi}\epsilon_{\mu\nu\tau\rho}\int_{-1}^{1}\hat{J}^\rho\left(x + \frac{s}{2}g'^2\hat{L}\right)ds\,, \tag{4.11}$$

where $\hat{J}^\rho$ is the U(1) current including the contribution of the scalars.

We see that the S-dual theory actually looks local order by order in g', and only appears non-local if we sum all orders in g'. In particular, the scale of non-locality in this description is $g'^2\hat{L}$. We can gain a clue as to the origin of the non-locality by rewriting (4.11) using (4.6)

$$F'_{\mu\nu}(x) = F_{\mu\nu}(x) - \frac{\hat{L}^\tau}{\pi}\epsilon_{\mu\nu\tau\rho}\frac{2\sin\frac{i}{2}g'^2\hat{L}\cdot\partial}{\frac{i}{2}g'^2\hat{L}\cdot\partial}\hat{J}^\rho(x)\,. \tag{4.12}$$

Notice that only even powers of g'^2 enter in the Taylor series expansion of

$$\frac{\sin\frac{i}{2}g'^2\hat{L}\cdot\partial}{\frac{i}{2}g'^2\hat{L}\cdot\partial}\,.$$

This suggests that the effect might be due to *closed* string interactions, as $g'^2 \approx g'_s$. It would be interesting to understand this behavior directly by studying string interactions in the S-dual RR plane wave background (1.4)–(1.6). Note that when the NSNS background (1.1)–(1.3) is transformed into the RR background (1.4)–(1.6) using S-duality, the Regge slope $\tilde{\alpha}'$ of (1.4)–(1.6) is given in terms of the Regge slope α' of (1.1)–(1.3) by $\tilde{\alpha}' = g_s\alpha'$. Using (3.3) and the definition of $\hat{L}$ in (4.9) we see that $\hat{L} = 2\pi\tilde{\alpha}'\mu$ and so is finite in the RR background.

In order to extend the discussion to N D3-brane probes we need to know the S-dual description of the dipole theory that is obtained as a deformation of $\mathcal{N} = 4$ Super Yang-Mills theory with gauge group U(N). Since the gauge fields that correspond to the U(1) center are IR free we can ignore them and consider only the SU(N) dipole theory. It is natural to conjecture that the dual of the lightlike SU(N) dipole theory is given by a prescription similar to (4.11)

$$
\begin{aligned}
F_{\mu\nu}(x) \rightarrow F_{\mu\nu}(x) - \frac{1}{\pi}\epsilon_{\mu\nu\tau\rho}\int_{-1}^{1} ds \left\{ \sum_{k=1}^{3} \hat{L}_k^\tau \hat{J}_k^\rho \left(x + \frac{s}{2}g'^2 \hat{L}_k\right) + \sum_{a=1}^{4} \hat{\lambda}_a^\tau \hat{J}_a^\rho \left(x + \frac{s}{2}g'^2\hat{\lambda}_a\right)\right\}, \\
\hat{J}_k^\mu \equiv \frac{1}{2}\left(i\overline{Z}_k D^\mu Z_k - iD^\mu \overline{Z}_k Z_k - iZ_k D^\mu \overline{Z}_k + iD^\mu Z_k \overline{Z}_k\right), \\
\hat{J}_a^\mu \equiv \frac{1}{2}\sigma^{\mu\alpha}{}_{\dot\alpha}\psi_\alpha^a \overline{\psi}_a^{\dot\alpha} - \frac{1}{2}\sigma^{\mu\alpha}{}_{\dot\alpha}\overline{\psi}_a^{\dot\alpha}\psi_\alpha^a
\end{aligned}
\tag{4.13}
$$

Here we used the notation of section 2 and we defined the rescaled dipole vectors of the bosons and fermions similarly to (4.9),

$$
\hat{L}_k^\tau \equiv \frac{1}{g'^2}L_k^\tau, \qquad \hat{\lambda}_a^\tau \equiv \frac{1}{g'^2}\lambda_a^\tau .
$$

The Lie algebra valued $\hat{J}_k^\mu$ and $\hat{J}_a^\mu$ are the individual contributions of the scalars and fermions to the $su(N)$ current. In (4.13) each contribution to the current enters with a coefficient that is proportional to the dipole vector of the corresponding field. We assume that the dipole vectors $\hat{L}_k$ and $\hat{\lambda}_a$ are all lightlike and pointing in the same direction. We also assume that (4.13) is written in the gauge $A_- = 0$. In this case all the residual gauge transformations are independent of x^- and (4.13) is gauge invariant.

5. Conclusion and discussion

In this paper we have argued that particular D-brane probes of plane-wave backgrounds are described by nonlocal field theories. In the case of an NSNS background we have identified the field theory as a lightlike dipole theory and we have verified the statement by an explicit lightcone string computation. In the case of an RR 3-form field strength background we have provided an indirect argument, using S-duality, for the nonlocality of the effective theory on D3-brane probes. This is a more complicated theory and we have conjectured the form of its lagrangian in (4.11). The nonlocality scale is proportional to g_s and it is obvious from (4.11) that one has to sum up contributions from all orders of string perturbation theory in order to exhibit the nonlocal nature of the interactions. It would be interesting to verify this directly from the solvable plane-wave string theory.[2] Note that, since the nonlocal interactions are in the lightlike direction, we can make the characteristic scale arbitrarily big by a coordinate transformation that rescales $x^\pm$. The excited string states can therefore decouple safely and, as the field theoretic S-duality suggests, the effective nonlocal field theory can be unitary.

It is interesting to extend these ideas to pp-wave backgrounds with other RR fluxes. For that purpose we adopt the following somewhat heuristic point of view. The dipole theories that we have described in section 2 have a correlation between R-symmetry charge

[2]Work in progress.

and electric flux. In the D3-brane language, every state with Spin(6) transverse angular momentum also behaves as a fundamental string of finite extent. The length of the string is proportional to its angular momentum and the proportionality constants are the dipole vectors $\vec{L}_k$. In the S-dual nonlocal theories that describe D3-brane probes in pp-waves with a 3-form RR flux every state with Spin(6) transverse angular momentum also behaves as a D1-brane of finite extent. We can extend this line of thought to other RR-backgrounds. For example, in a background with a 5-form RR field strength $F_{+1234} = F_{+5678}$ (where we use lightcone coordinates $+, -, 1, \ldots, 8$ as in [5]) and a D5-brane in directions $+, -, 1235$ we should find that open string states attached to the D5-brane that have, say, angular momentum in the 78 plane also behave as a D3-brane that is spread in directions $+, -, 56$ and has a finite volume that is proportional to the angular momentum. This statement is, admittedly, obscure and it would be interesting to elucidate such a theory further.

Another possible application of the ideas presented in this paper is to M(atrix)-theory [40]. The M(atrix)-theory hamiltonian for M-theory is the 0+1D supersymmetric Yang-Mills quantum mechanics [41]–[44]. The standard derivation of weakly coupled type-IIA string theory from M(atrix)-theory [45]–[47] requires understanding of the strong coupling limit of 1+1D $\mathcal{N} = 8$ Super Yang-Mills theory. Dipole theories naturally appear as M(atrix) models of Melvin spaces [28] (see also [26, 27]). The relevant M(atrix) models are dipole theories that are deformations of 1+1D $\mathcal{N} = 8$ Super Yang-Mills theory. Therefore understanding the strong coupling limit of dipole theories could prove beneficial for deriving a weakly coupled string theory descriptions of Melvin spaces. (A string theory description for Melvin backgrounds has been given in [48] but it has a dilaton that is not bounded.) Perhaps nonlocal worldsheet theories will play a role in such a description. (See [49]–[51] for other ideas regarding nonlocal worldsheet theories.)

We would also like to mention another new kind of nonlocal theory that appears on D3-brane probes in certain backgrounds with strong NSNS flux [52]–[54]. It is a very intriguing nonlocal field theory that is not translationally invariant and is described as a gauge theory on a noncommutative space with a varying noncommutativity parameter.

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